THE CHINESE UNIVERSITY OF HONG KONG

DEPARTMENT OF MATHEMATICS

MATH2010D Advanced Calculus 2019-2020

Errata and Remarks for Lecture Notes (Updated on 9 Apr, 2020)

1. Part 1 P.19 Example 3.1: To be precise, $r \geq 0$ and $\theta = \theta_0$ defines a ray.

2. Part 1 P.22: To be more explicit:

$$\begin{aligned} \text{Areas of } i\text{-th triangle} &= \frac{1}{2} r_{i+1} r_i \sin(\theta_{i+1} - \theta_i) \\ &= \frac{1}{2} r_{i+1} r_i \sin \Delta\theta \\ &\approx \frac{1}{2} r_i^2 \Delta\theta \end{aligned}$$

   We have the last line since $r_{i+1} \approx r_i$ and $\lim_{\Delta\theta \to 0} \frac{\sin \Delta\theta}{\Delta\theta} = 1$ which implies $\sin \Delta\theta \approx \Delta\theta$.

3. Part 1 P. 24: For the change of coordinates $(\rho, \phi, \theta) \to (x, y, z)$, the last equation should be "$z = \rho \cos \phi$".

4. Part 2 P.1 Definition 4.2: It should be "
   (1) $\vec{x_0}$ is said to be ...
   (2) $\vec{x_0}$ is said to be ...
   (3) $\vec{x_0}$ is said to be ...

5. Part 2 P.2 Definition 4.4: It should be "$\vec{x_0} \in \mathbb{R}^n$ is said to be a cluster point of S if for all $r > 0$, $B_r^0(\vec{x_0}) \cap S \neq \phi$".

6. Part 2 P.4 Definition 4.6: It should be "S is said to be path connected if ...".

   Generally speaking, path connectedness and connectedness are different in topology. Path connectedness implies connectedness, but the converse is not ture. However, any open connected subset S in $\mathbb{R}^n$ is path connected. Therefore, path connectedness and connectedness are equivalent for open subsets in $\mathbb{R}^n$.

7. Part 2 P.8 Example 5.7: It should be "..., where $A \in M_{3\times 2}(\mathbb{R})$, $\vec{x} \in M_{3\times 1}(\mathbb{R})$ and $\vec{b} \in M_{2\times 1}(\mathbb{R})$."

8. Part 3 P.3 Example 6.2: It should be "$|x| \leq \sqrt{x^2 + y^2} < \delta$".

9. Part 3 P.4 Exercise 6.1: It should be "ϵ-δ definition". In third line of the proof, it should be "$|x| \leq \sqrt{x^2 + y^2} < \delta$ and $|y| \leq \sqrt{x^2 + y^2} < \delta$".

10. Part 3 P.6 Proposition 6.2: It should be "$\lim_{\vec{x} \to \vec{x_0}} f(\vec{x}) = L$ if and only if ...".

11. Part 3 P.8 Example 6.8: It should be "$\lim_{t\to 0} (f \circ \gamma_1)(t) = \cdots = \lim_{t\to 0} \frac{mt^3}{t^4 + m^2 t^2} = \lim_{t \to 0} \frac{mt}{t^2 + m^2} = 0$".

12. Part 4 P.5 Example 8.4: It should be "$\cdots = \lim_{h\to 0} \frac{2}{5}x + \frac{1}{\sqrt{5}} y + \frac{2}{5} h = \frac{2}{5} x + \frac{1}{\sqrt{5}} y$".

13. Part 4 P.5 Definition 8.3: Suppose that $f : D \to \mathbb{R}$ is a C^1 function, where D is an open subset in $\mathbb{R}^n$. I only mentioned that all first partial derivatives of f are continuous on D. However, you will see later that all first partial derivatives of f are continuous on D implies that f is a differentiable on D (Theorem 9.2), and then implies f is continuous D (Theorem 9.3).

14. Part 4 P.6 Proof of Theorem 8.1: It should be

$$\begin{aligned}
\frac{\partial^2 f}{\partial x \partial y}(a,b) &= \lim_{h\to 0}\lim_{k\to 0}\left(\frac{[f(a+h,b+k)-f(a+h,b)]-[f(a,b+k)-f(a,b)]}{h}\right)\cdot\frac{1}{k} \\
&= \lim_{h\to 0}\lim_{k\to 0}\frac{\frac{\partial f}{\partial x}(c_1,b+k)-\frac{\partial f}{\partial x}f(c_1,b)}{k} \\
&= \lim_{h\to 0}\lim_{k\to 0}\frac{\partial^2 f}{\partial y \partial x}(c_1,c_2) \\
&= \lim_{h\to 0}\frac{\partial^2 f}{\partial y \partial x}(c_1,b) \\
&= \frac{\partial^2 f}{\partial y \partial x}(a,b)
\end{aligned}$$

15. Part 4 P.7 Exercise 9.1: The third equation should be $\nabla(\frac{f}{g})(\overrightarrow{x_0}) = \dfrac{g(\overrightarrow{x_0})\nabla f(\overrightarrow{x_0}) - f(\overrightarrow{x_0})\nabla g(\overrightarrow{x_0})}{[g(\overrightarrow{x_0})]^2}$

16. Part 4 P.18 Total Differential: It should be "$\Delta f \sim \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(\overrightarrow{x_0})\Delta x_i$" and "$df = \sum_{i=1}^{n}\frac{\partial f}{\partial x_i}(\overrightarrow{x_0})dx_i$".

17. Part 4 P.19: Please remove the remark "row vector in $\mathbb{R}^n$" and in fact, it is a real number.

18. Part 4 P.20 Theorem 9.4: To be precise, we have the following:

    Let $D \subset \mathbb{R}^n$ be an open subset, let $\overrightarrow{x_0} \in D$ and let $f : D \to \mathbb{R}^m$.

    (a) f is differentiable at $\overrightarrow{x_0}$ if and only if each f_i is differentiable at $\overrightarrow{x_0}$.

    (b) Furthermore, if f is differentiable at $\overrightarrow{x_0}$, then

    $$Df(\overrightarrow{x_0}) = \begin{bmatrix} \nabla f_1(\overrightarrow{x_0}) \\ \vdots \\ \nabla f_m(\overrightarrow{x_0}) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(\vec{x}_0) & \cdots & \frac{\partial f_1}{\partial x_n}(\vec{x}_0) \\ \vdots & & \vdots \\ \frac{\partial f_m}{\partial x_1}(\vec{x}_0) & \cdots & \frac{\partial f_m}{\partial x_n}(\vec{x}_0) \end{bmatrix} \in M_{m\times n}(\mathbb{R}),$$

    which is called total derivative of f at $\overrightarrow{x_0}$, is the unique matrix such that

    $$\lim_{\overrightarrow{h}\to\overrightarrow{0}} \frac{|f(\overrightarrow{x_0}+\overrightarrow{h}) - (f(\overrightarrow{x_0}) + Df(\overrightarrow{x_0})\cdot\overrightarrow{h})|}{\overrightarrow{h}} = 0.$$

19. Part 4 P.21 Example 9.8: It should be "$\begin{bmatrix} 2\cos z & -\cos z & -(2x-y+1)\sin z \\ e^x\sin(2y+z) & 2e^x\cos(2y+z) & e^x\cos(2y+z)\end{bmatrix}$".

20. Part 6 P.5 Example 13.4: It should be "f is strictly decreasing from $(0,0)$ to $(2,4)$", "f is strictly increasing from $(2,4)$ to $(4,8)$" and "Global min of $f = -8$". Also, the saddle point is $(3,2)$.

21. Part 6 P.9 Theorem 14.1: It should be "γ lies on $L_c(g)$".

More mistakes may be found and more remarks may be added later, so this file will be updated accordingly.